Comments

Comments on "How Invariant is the Measured Equation of Invariance?"

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***Abstract*— In their paper appeared in the Feb. 1995 issue of this journal, the authors Jevtić and Lee claimed that they proved the measured equation of invariance (MEI) [1] to be not invariant to excitation. This letter points out the defect of their proof.**

I. Introduction

In the above paper,[1] the authors claimed that they proved the MEI [1] to be not invariant to excitation. Their paper is a follow up of an earlier paper [2], which also claimed to have proved the noninvariance of MEI. We have showed that the proof of their earlier paper to be incorrect and the data misinterpreted [3]. In this paper we show that their new proof is again incorrect, and the defect in their proof to be elementary and obvious.

The so-called proof in the paper is a counter example showing that two different sets of metrons produce a different set of MEI coefficients. But, the authors fail to observe that in the process of calculating the MEI equations, the two different calculations should reach the same order of accuracy. To be specific, the authors used the asymptotic form of the Hankel's function to calculate the MEI coefficients using $e^{-2j\phi}, e^{-j\phi}, 1, e^{j\phi}, e^{j2\phi}$ as metrons and obtain

$$\left.\begin{aligned} a_1 &= -\frac{1}{2}\frac{3+(\alpha_r k\Delta\rho)^2+j3k\Delta\rho}{3-5(\alpha_r k\Delta\rho)^2+j3k\Delta\rho} \\ a_2 &= \frac{1}{2}e^{-jk\Delta\rho}\frac{3+(\alpha_r k\Delta\rho)^2-j3k\Delta\rho}{3-5(\alpha_r k\Delta\rho)^2+j3k\Delta\rho} \\ a_3 &= -e^{-jk\Delta\rho}\frac{3-5(\alpha_r k\Delta\rho)^2-j3k\Delta\rho}{3-5(\alpha_r k\Delta\rho)^2+j3k\Delta\rho} \end{aligned}\right\} + O\left(\frac{1}{kb}\right) \tag{1}$$

where α_r stands for the aspect-ratio of the FD cell defined by

$$\alpha_r = \frac{b\Delta\phi}{\Delta\rho}.$$

The differential equation they recover from the FD equation is

$$L = \frac{\partial}{\partial(kr)} + j + j\frac{3}{4(kb)^2}\frac{\partial^2}{\partial\phi^2} + \frac{1}{4(kb)^2}\frac{\partial^3}{\partial(k\rho)\partial\phi^2}. \tag{2}$$

Using the asymptotic form of Hankel's function again to calculate MEI coefficients using$e^{-jkb\phi}, e^{-j\phi}, 1, e^{j\phi}, e^{jkb\phi}$ as metrons the authors obtain the following MEI coefficients and differential equation:

$$\left.\begin{aligned} a_1 &= -\frac{1}{2}\frac{1-\cos(\alpha_r k\Delta\rho)+j\alpha_r^2 k\Delta\rho(e^{jk\Delta\rho}-1)}{1-\cos(\alpha_r k\Delta\rho)+j\alpha_r^2 k\Delta\rho(e^{jk\Delta\rho}-1)\cos(\alpha_r k\Delta\rho)} \\ a_2 &= \frac{1}{2}\frac{1-\cos(\alpha_r k\Delta\rho)+j\alpha_r^2 k\Delta\rho(1-e^{-jk\Delta\rho})}{1-\cos(\alpha_r k\Delta\rho)+j\alpha_r^2 k\Delta\rho(e^{jk\Delta\rho}-1)\cos(\alpha_r k\Delta\rho)} \\ a_3 &= -\frac{1-\cos(\alpha_r k\Delta\rho)+j\alpha_r^2 k\Delta\rho(1-e^{-jk\Delta\rho})\cos(\alpha_r k\Delta\rho)}{1-\cos(\alpha_r k\Delta\rho)+j\alpha_r^2 k\Delta\rho(e^{jk\Delta\rho}-1)\cos(\alpha_r k\Delta\rho)} \end{aligned}\right\} + O\left(\frac{1}{(kb)^{1/3}}\right) \tag{3}$$

$$\mathbf{L} = \frac{1}{2}\left[\frac{\partial}{\partial(k\rho)} + j + j\frac{1}{(kb)^2}\frac{\partial^2}{\partial\phi^2} + \frac{1}{2(kb)^2}\frac{\partial^3}{\partial(k\rho)\partial\phi^2}\right]. \tag{4}$$

Since the differential equations are not the same, they claim their proof of noninvariance. But, the a_i's of (1) and (3) are of different order of accuracy. Equation (1) is of $O(\frac{1}{kb})$ and (3) is of $O(\frac{1}{(kb)^{1/3}})$. So, they cannot be the same. They should add more terms to (3) to bring it to the same order of accuracy as (1). Those missing terms could contribute to the difference in the differential equations, so the proof of noninvariance is invalid.

We consider the defect in the above paper to be very elementary and obvious, and it was communicated to the authors through the reviewing process. The fact this paper is published without answering to the reviewers criticism is of great concern to us.

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[1] J. O. Jevtic and R. Lee, *IEEE Microwave and Guided Wave Lett.*, vol. 5, no. 2, pp. 45–47, Feb. 1995.

References

[1] K. K. Mei, R. Pous, Z. Chen, Y. W. Liu, and M. Prouty, "Measured equation of invariance: A new concept in field computation," *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 320–328, Mar. 1994.

[2] J. O. Jevtic and R. Lee, "A theoretical and numerical analysis of the measured equation of invariance," *IEEE Trans. Antennas Propagat.*, vol. 42, no. 8, pp. 1097–1105, Aug. 1994.

[3] K. K. Mei and Y. Liu, "Comments on 'A theoretical and numerical analysis of the measured equation of invariance,'" to be published in *IEEE Trans. Antennas Propagat.*, Oct. 1995.

Authors' Reply by Jovan O. Jevtić and Robert Lee

Let us consider (1) and (3) in the comment by Mei, which are the same as (5) and (11) in our original letter. Two different sets of MEI coefficients, given by these two equations, correspond to two different sets of metrons. Both expressions are asymptotic expansions in terms of the electrical radius kb of the grid boundary. Only the first, most significant terms are given explicitly and they, naturally, do not depend on kb but only on the electrical size of the finite difference grid cell, which is $k\Delta p$ by $\alpha_r k\Delta p$. For the remainder of the asymptotic expansions, we have only specified the order of the leading terms.

Dr. Mei is concerned that the order of these remaining terms are not the same in both equations, and that consequently, differential equations (2) and (4) in the comment, which are the same as (9) and (12) in the original paper, cannot be compared due to the terms that are supposedly missing.

We fail to see how the higher-order terms in an asymptotic expansion could make it equal to another asymptotic expansion, if the first, most significant terms are not equal. Thus, the MEI coefficients given by (1) and (3) must be different. Furthermore, since we have obtained the differential operators (2) and (4) through a linear combination of the MEI coefficients, the higher-order terms will remain the higher-order terms, unless all of the most significant terms cancel out, which is clearly not the case. The higher-order terms can be made arbitrarily small by selecting a sufficiently large kb. Thus, (2) and (4) represent the first, most significant term in the asymptotic expansion for the boundary operators.

To conclude, we showed that there are no missing terms in (2) and (4) for the equivalent differential operators. We therefore stand by the conclusion we made in our original letter.

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